Math 226 - Fall 2017	Due Friday, October 6th

Homework 6

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Book problems in (parentheses) are suggested. Book problems in **bold** will be turned in, along with the additional problem below.

Section 11.5 # (1), **2**, (7), **8**, (9), **10**, (11), **12**, (13), **14**, (17), **18**, (23), (27), **28**, (31), **32**, (36), (41), **42**, (45), **46**, (47)

Section 11.6 # (1), **2**, (3), **4**, (7), **8**, (13), **14**, (15), **16**, (19), **20**, (25), **26**, (29), **30**, (31), **32**, (35), (36), (39), **40**, (41), **42**, (43), **44**, (45), **46**, (47), (51), **52(a)**, (53), **54**

1. Suppose a particle of mass m moves along a twice-differentiable path $\vec{x}(t)$ satisfying the differential equation

$$m\vec{x}''(t) = -\nabla V(\vec{x}(t)),$$

where $V\colon \mathbb{R}^n \to \mathbb{R}$ is a differentiable function. Let

$$E(t) = \frac{1}{2}m|\vec{x}'(t)|^2 + V(\vec{x}(t)).$$

Prove that $E(t)$ is constant. (This is the standard conservation of energy law for motion under the influence of a potential V.)

2. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. Let us write $f = f(u, v)$, and let $g(x, y) = f(x + y, x - y)$. Prove that

$$\frac{\partial g}{\partial x}\frac{\partial g}{\partial y} = \left(\frac{\partial f}{\partial u}\right)^2 - \left(\frac{\partial f}{\partial v}\right)^2.$$

3. Show that at any point where the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is smooth, we have the identity

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial g}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial g}{\partial \theta}\right)^2$$

where $g(r, \theta) = f(r\cos\theta, r\sin\theta)$, (x, y) are standard Cartesian coordinates, and (r, θ) are polar coordinates.

4. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the function $f(x, y)$ which satisfies

$$f(r\cos\theta, r\sin\theta) = r + \sin(\theta), \quad r \neq 0.$$

Using the result of the previous problem, find the maximum value of $D_{\vec{u}}f(2, 0)$, where $\vec{u}$ is an arbitrary unit vector.

(Hint: What can you say about $|\nabla f(2, 0)|$?)