Last time:

- Ways of representing functions $y = f(x)$ (of a single variable)
  - graph
  - table
  - formula
- linear functions
- average rate of change

Today:

- Applications to business / economics
  - cost, revenue, profit functions
  - supply + demand curves
- Exponential growth / decay models

Ex

Annual sales of music compact discs (CDs) have declined since 2000. Sales were 942.5 million in 2000 and 384.7 million in 2008.[17]

(a) Find a formula for annual sales, S, in millions of music CDs, as a linear function of the number of years, t, since 2000.

(b) Give units for and interpret the slope and the vertical intercept of this function.

(c) Use the formula to predict music CD sales in 2012.

Want a formula for a linear function $S = f(t) = \underbrace{b+mt}_{\text{linear model}}$

so that:

- When $t=0$, $S=942.5$ $\rightsquigarrow$ $b = 942.5$
- When $t=8$, $S=384.7$

$$m = \text{slope} = \frac{\Delta S}{\Delta t} = \frac{384.7-942.5}{8-0} = \frac{-557.8}{8} = -69.725$$

a) $\boxed{S = f(t) = 942.5 - (69.725)t}$

units ↓

b) b = number of CDs sold in 2000 (in millions of CDs)

m = amount sales change each year (in millions of CDs per year)

c) Model predicts that when $t=12$, we have:

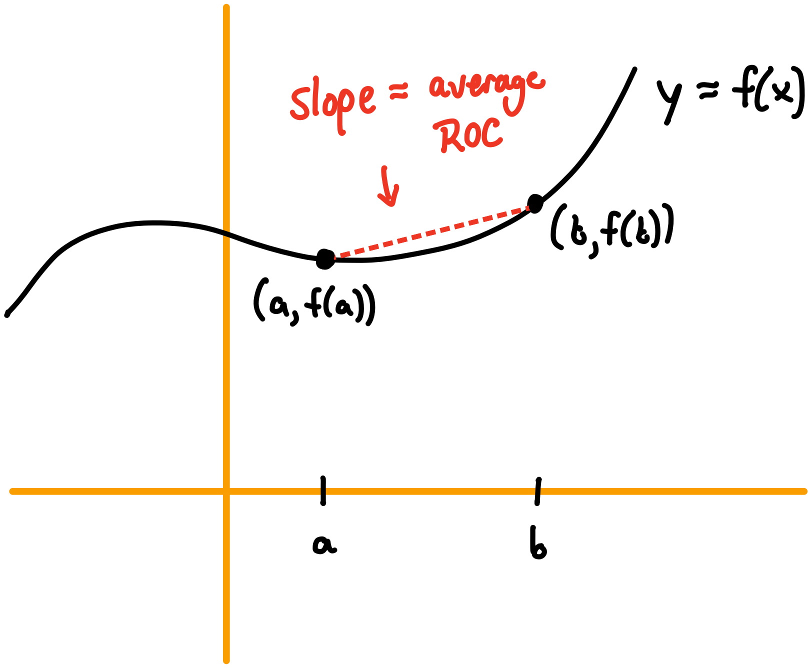
$$S = f(12) = 942.5 - (69.725)\cdot 12$$

$$\approx \boxed{105.8} \quad \text{(millions of CDs)}$$

Average rate of change (ROC) : $\boxed{\dfrac{f(b)-f(a)}{b-a} = \dfrac{\Delta y}{\Delta x}}$

(of a function $y=f(x)$ over the interval $[a,b]$)

equals the slope of the line segment connecting the two points $(a,f(a))$ and $(b,f(b))$ (the "secant line")

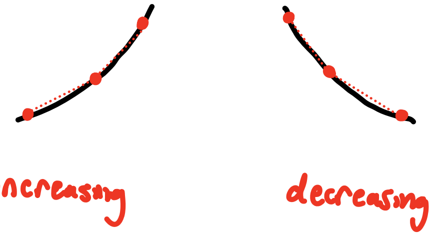


Note : When a,b aren't too far apart, this line segment seems to approximate the graph pretty well

⤳ This observation will be the basis for essentially all of calculus.

Concavity

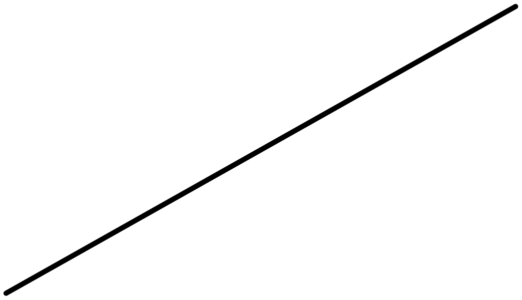
concave upward :



concave downward :

Linear functions $y = f(x) = b + mx$ $\frac{\Delta y}{\Delta x} = m$ (always)

↳ Average rate of change always the same $(= m)$



Ex The population of a city in thousands is shown across several years:

			+2		+9		+7	
t	Year	1992		1994		2005		2012
P	Pop.	193		215		275		292
			+22		+60		+17	

Average rate of change: $\frac{\Delta P}{\Delta t}$

1992 - 1994: $\frac{\Delta P}{\Delta t} = \frac{215 - 193}{1994 - 1992} = \frac{22}{2} = 11$

↳ population was increasing by 11,000 people per year on average, between 1992 and 1994

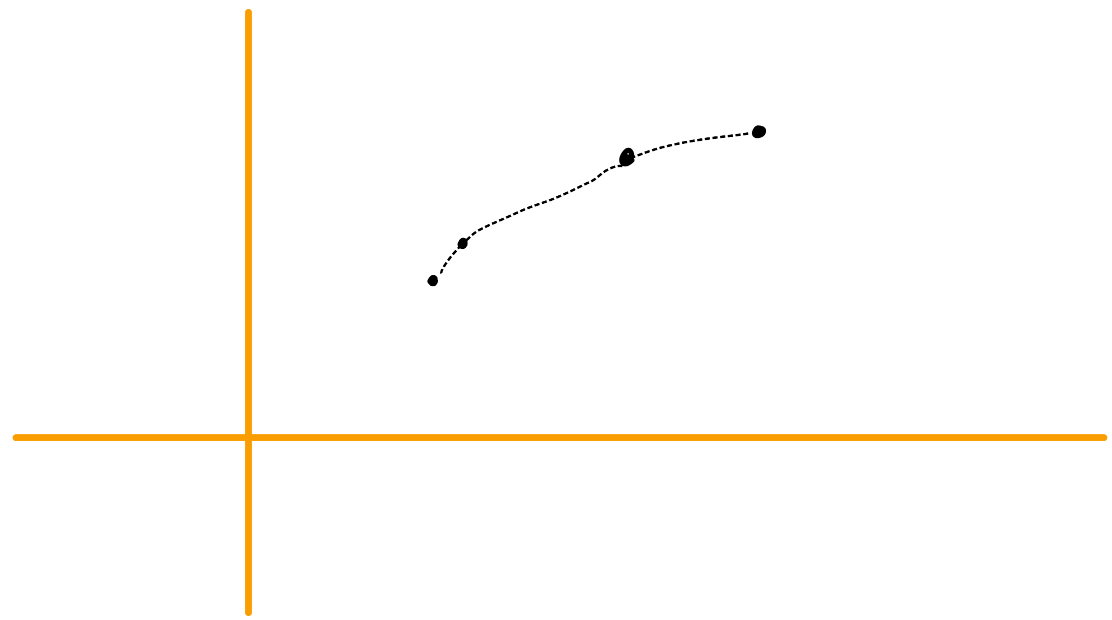
1994 - 2005: $\frac{\Delta P}{\Delta t} = \frac{275 - 215}{2005 - 1994} = \frac{60}{11} \approx 5.45$

↳ population was increasing by ≈ 5,450 people per year on average, between 1994 and 2005

2005 - 2012: $\frac{\Delta P}{\Delta t} = \frac{292-275}{2012-2005} = \frac{17}{7} \approx 2.43$

↳ population was increasing by ≈ 2,430 people per year on average, between 2005 and 2012

⇝ data appears to be concave downward



Applications to business/economics

Common functions we will analyze

cost function : cost of producing q units

$C = C(q)$

revenue function : revenue from selling q units

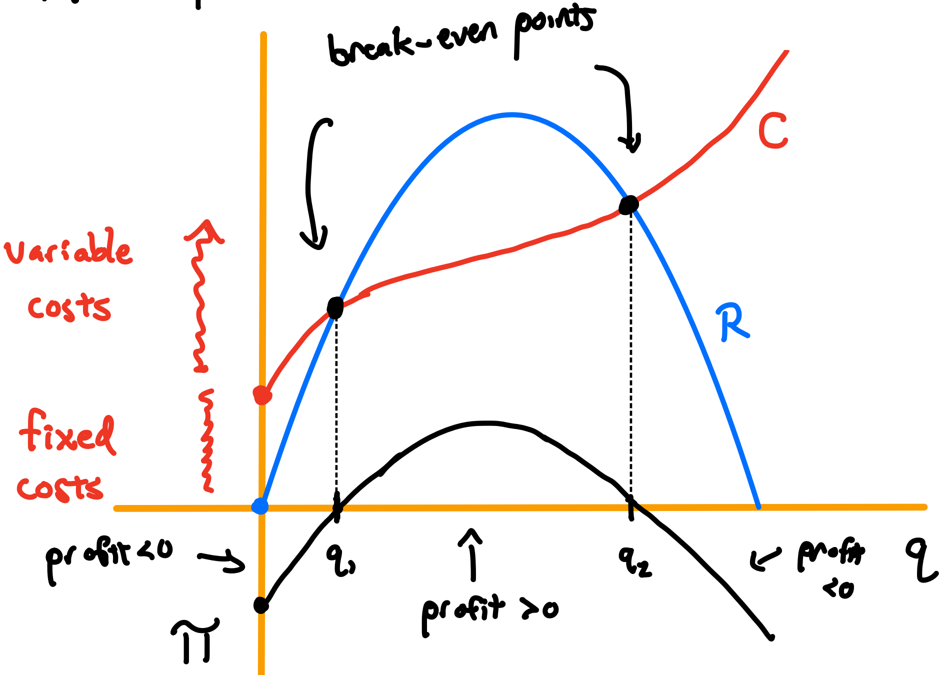
$R = R(q)$

↳ sometimes in the presence of a demand curve

profit function : profit from selling q units

$$\underset{\displaystyle \Pi}{\underset{\shortparallel}{\pi}} = \pi(q) = R(q) - C(q)$$

Typical picture:



$$\pi(q_1) = 0$$

$$\pi(q_2) = 0$$

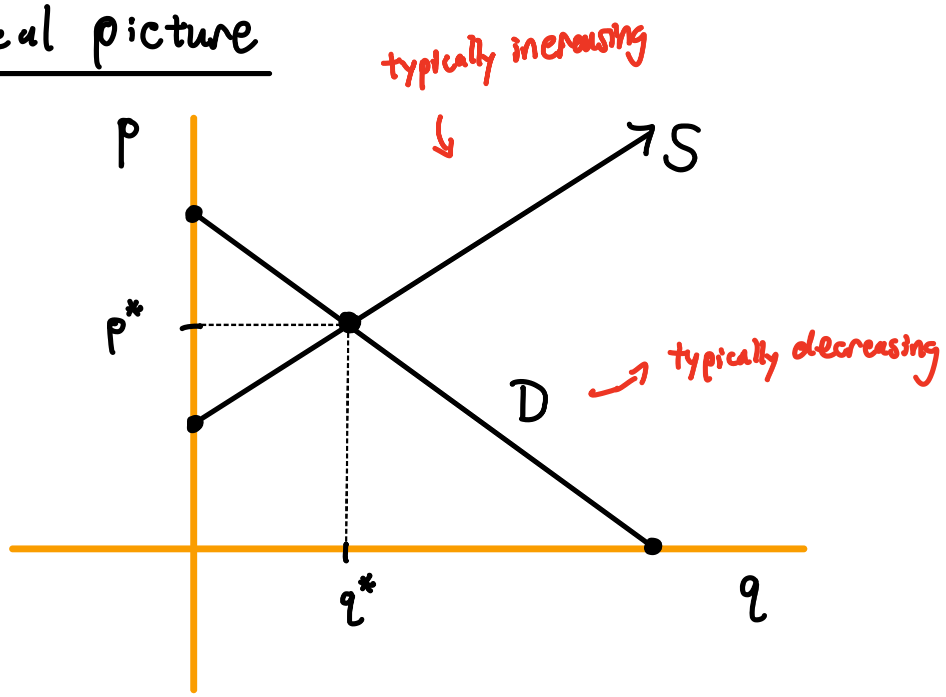
Supply/demand curves

Supply curve : determines the quantity q supplied when the price is p

demand curve : determines the quantity q demanded when the price is p

equilibrium price/quantity : the values of p, q where the supply + demand curves meet

Typical picture



Ex The cost of producing widgets is \$10/unit.

S No one will produce them at cost, but for each \$1 above \$10, an additional 30 units (per month) will be supplied.

D When the price is \$10, the quantity demanded is 600 units (per month), and for each \$1 increase in price the demand is reduced by 20 units. (per month)

What is the equilibrium price/quantity (p^*, q^*)?

S: $q = 0$ when $p = 10$ → $(0, 10)$ point

$$\uparrow$$

$$(q_0, p_0)$$

$$\frac{\Delta q}{\Delta p} = \frac{30}{1} = 30 = m$$

$$y - y_0 = m(x - x_0)$$

$$q - q_0 = m(p - p_0)$$

$$q - 0 = 30(p - 10)$$

$$\boxed{q = 30p - 300}$$

D: $q = 600$ when $p = 10$

$(600, 10)$

(q_1, p_1)

$$\frac{\Delta q}{\Delta p} = \frac{-20}{1} = -20 = n$$

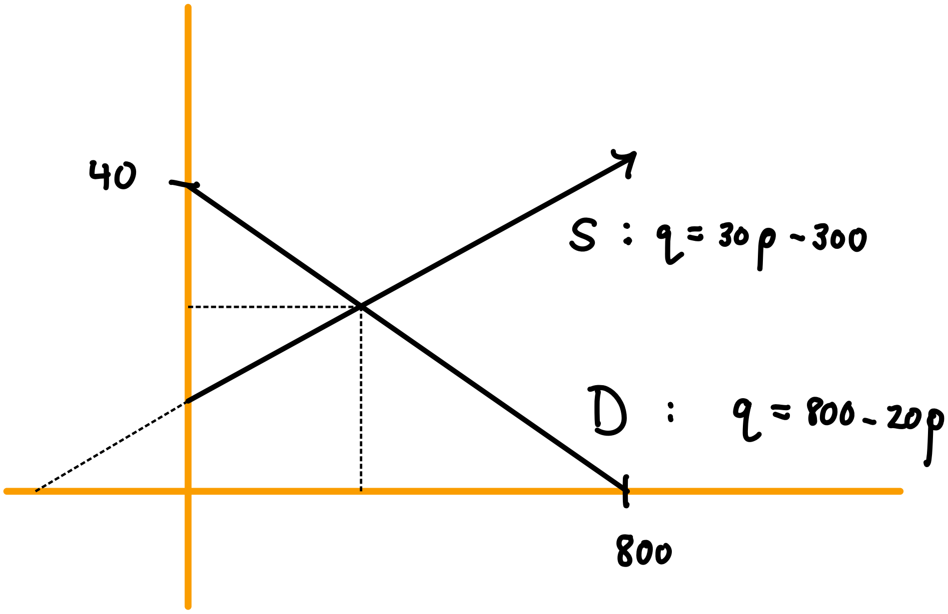
$$q - q_1 = n(p - p_1)$$

$$q - 600 = -20(p - 10)$$

$$q = 600 - 20(p - 10)$$

$$= 600 - 20p + 200$$

$$\boxed{q = 800 - 20p}$$



$$\left.\begin{array}{ll} S: & q = 30p - 300 \\ D: & q = 800 - 20p \end{array}\right\}$$

Equilibrium : Intersection

$$30p - 300 = 800 - 20p$$

$$50p = 1100$$

$$p = 22$$

$$q = 30(22) - 300 = 660 - 300 = 360$$

$$q = 800 - 20(22) = 800 - 440 = 360 \quad \checkmark$$

$$\boxed{(q^*, p^*) = (360, 22)}$$